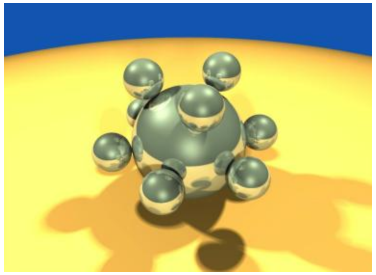
Computer Graphics using OpenGL, 3rd Edition
F. S. Hill, Jr. and S. Kelley



Chapter 6.1-3
Modeling Shapes with Polygonal Meshes

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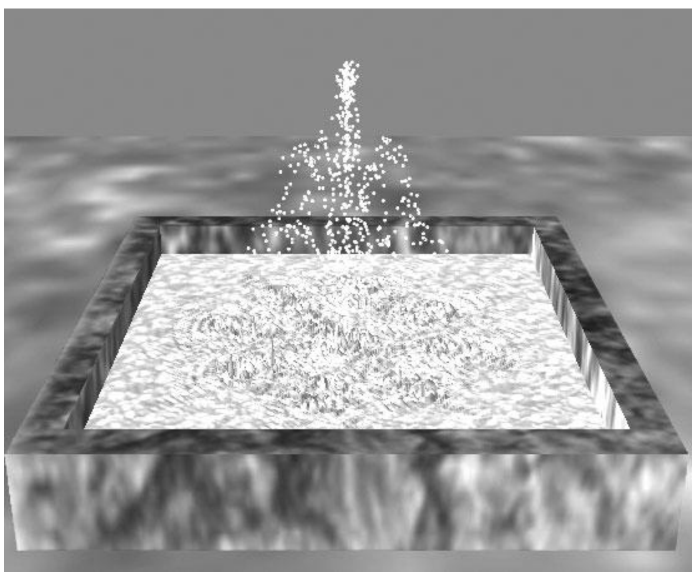
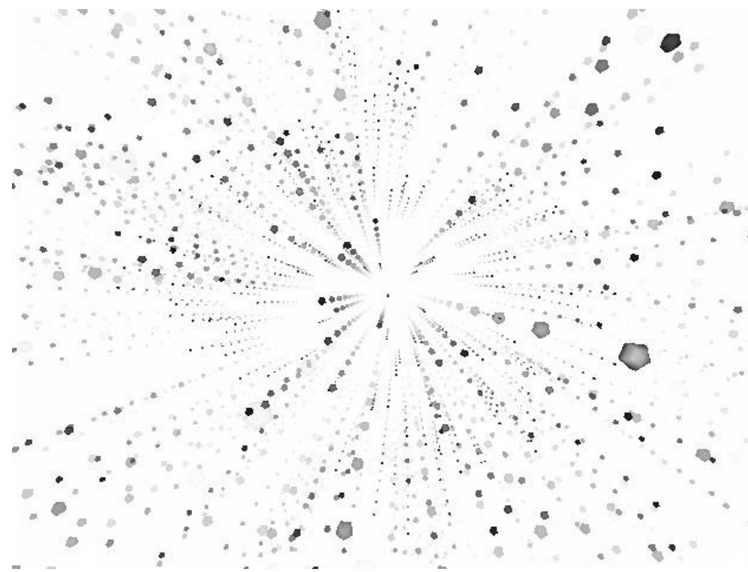
3D Modeling

- Polygonal meshes capture the shape of complex 3D objects in simple data structures.
  - Platonic solids, the Buckyball, geodesic domes, prisms.
  - Extruded or swept shapes, and surfaces of revolution.
  - Solids with smoothly curved surfaces.
- Animated Particle systems: each particle responds to conditions.
- Physically based systems: the various objects in a scene are modeled as connected by springs, gears, electrostatic forces, gravity, or other mechanisms.

Particle Systems Example

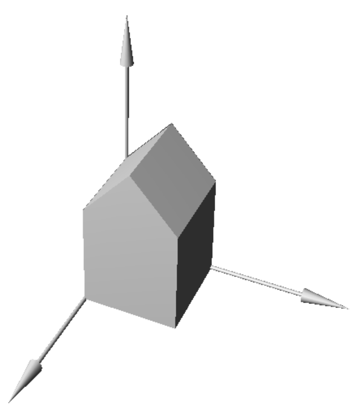
- **Particle system showing water droplets in a fountain. (Courtesy of Philipp Crocoll); Starfield simulation (Courtesy of Ge Wang)**

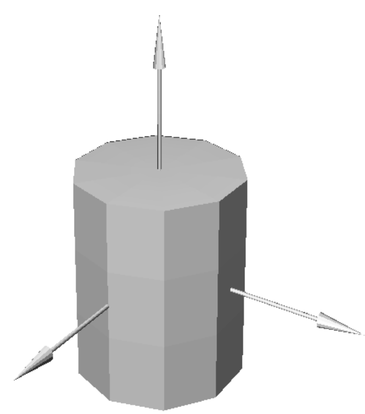




Polygonal Meshes

- A polygonal mesh is a collection of polygons (faces) that approximate the surface of a 3D object.
  - Examples: surfaces of sphere, cone, cylinder made of polygons (Ch. 5); barn (below).





Polygonal Meshes (2)

- Polygons are easy to represent (by a sequence of vertices) and transform.
- They have simple properties (a single normal vector, a well-defined inside and outside, etc.).
- They are easy to draw (using a polygon-fill routine, or by mapping texture onto the polygon).

Polygonal Meshes (3)

- Meshes are a standard way of representing 3D objects in graphics.
- A mesh can approximate the surface to any degree of accuracy by making the mesh finer or coarser.
- We can also smooth the polygon edges using rendering techniques.

Polygonal Meshes (4)

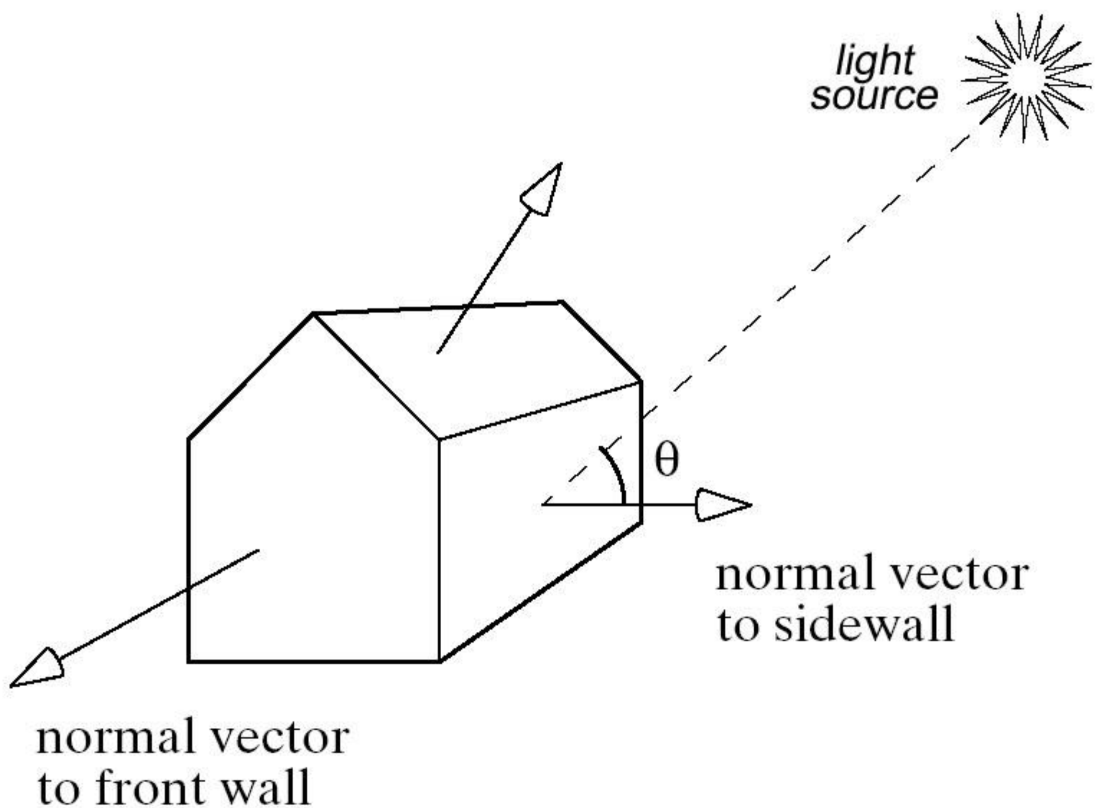
- Meshes can model both solid shapes and thin skins.
  - The object is **solid** if the polygonal faces fit together to enclose space.
  - In other cases, the faces fit together without enclosing space, and so they represent an infinitesimally thin surface.
- In both cases we call the collection of polygons a **polygonal mesh** (or simply a **mesh**).

Polygonal Meshes (5)

- A polygonal mesh is described by a list of polygons, along with information about the direction in which each polygon is facing.
- If the mesh represents a solid, each face has an inside and an outside relative to the rest of the mesh.
- In such a case, the directional information is often simply the outward pointing **normal vector** to the plane of the face used in the shading process.

Polygonal Meshes (6)

- The normal direction to a face determines its brightness.

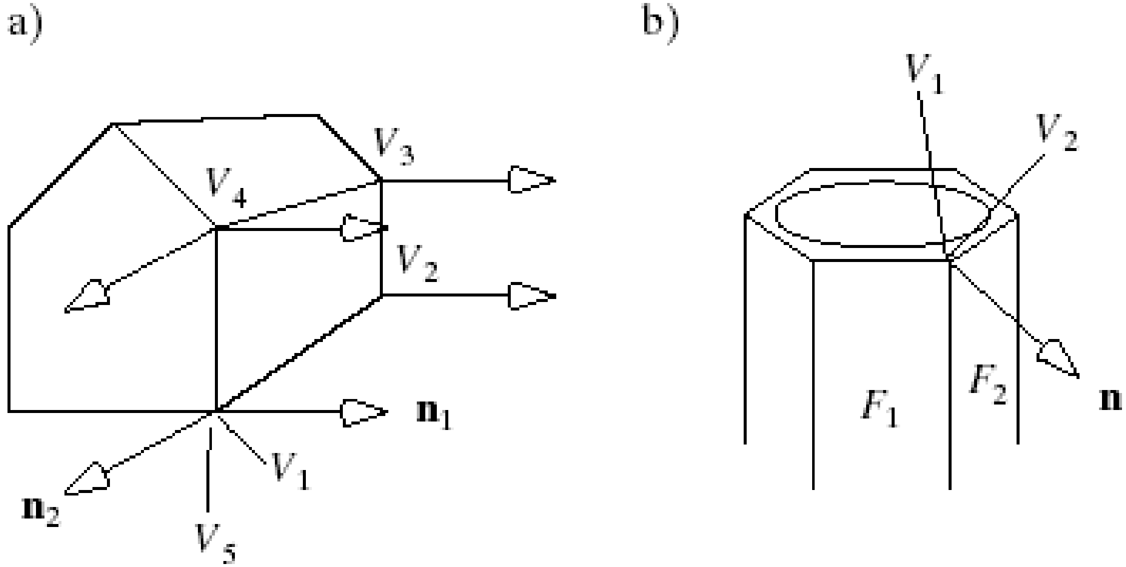


Polygonal Meshes (7)

- For some objects, we associate a *normal vector* to <u>each</u> <u>vertex</u> of a face rather than one vector to an entire face.
  - We use meshes, which represent objects with smoothly curved faces such as a sphere or cylinder. We will refer to the faces of such objects, but with the idea that there is a "*smooth-underlying surface*".
  - When we display such an object, we will want to de-emphasize the individual faces of the object in order to make the object look smooth.

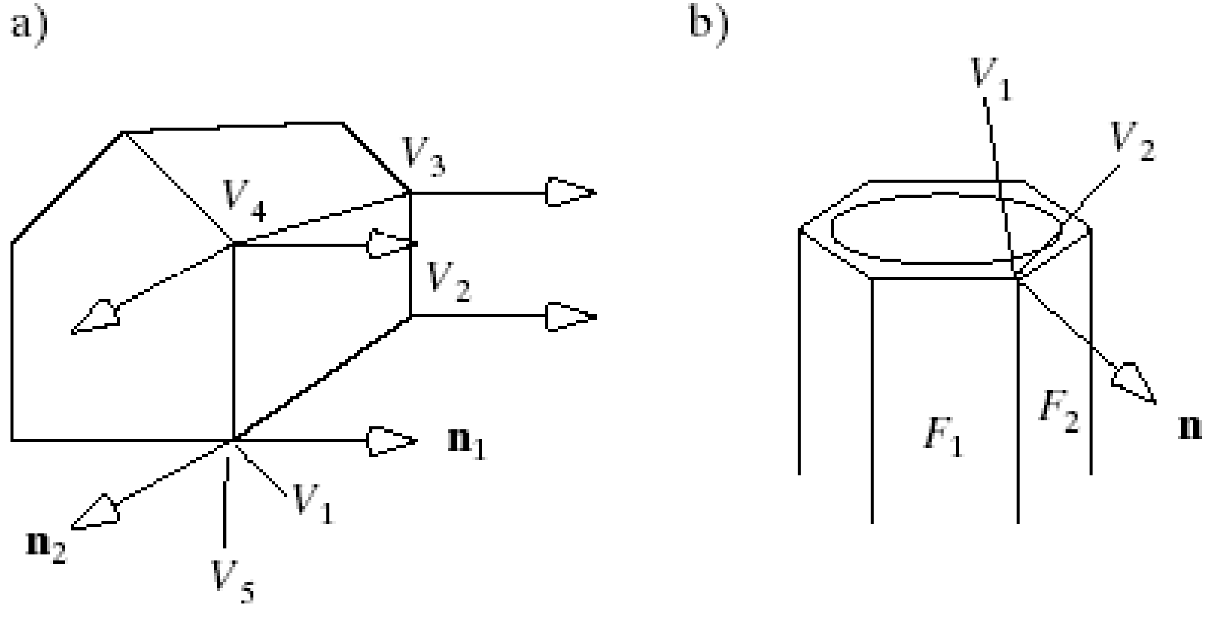
Polygonal Meshes (8)

- Each vertex V_1, V_2, V_3, and V_4 defining the side wall of the barn has the *same* normal $\mathbf{n}_1$, the normal vector to the side wall.
- But vertices of the front wall, such as V_5, will use normal $\mathbf{n}_2$. (Note that vertices V_1 and V_5 are located at the same point in space, but use different normals.)



Polygonal Meshes (9)

- For the smoothly curved surface of the cylinder, both vertex V_1 of face F_1 and vertex V_2 on face F_2 use the same normal ***n***, the vector perpendicular to the underlying smooth surface.

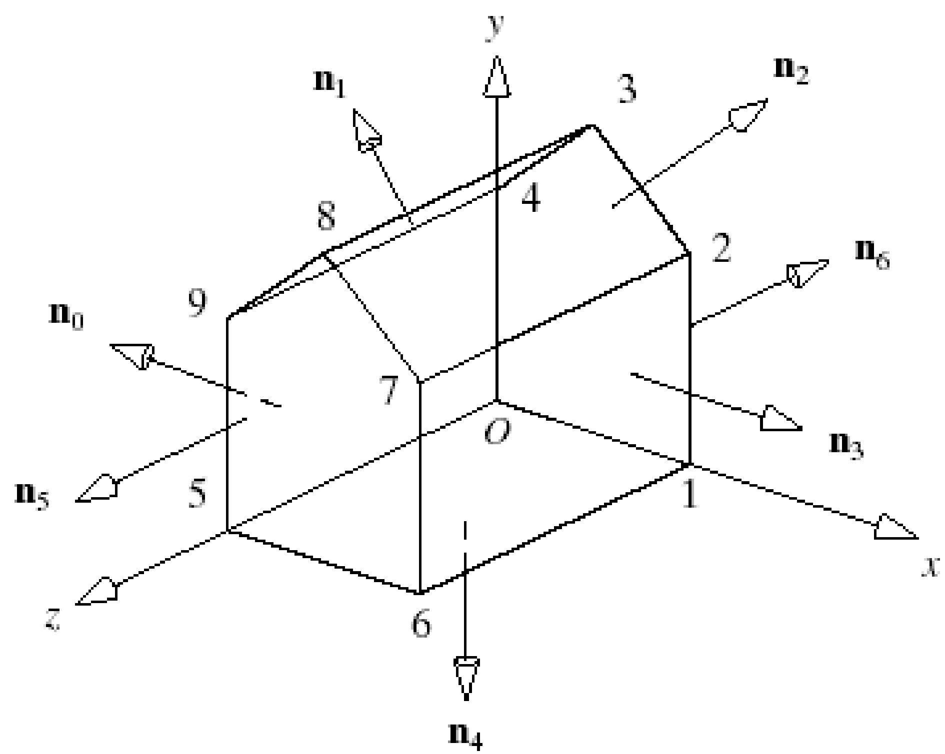


Defining a Polygonal Mesh

- A mesh consists of 3 lists: the vertices of the mesh, the outside normal at each vertex, and the faces of the mesh.
- Example: the basic barn has 7 polygonal faces and 10 vertices (each shared by 3 faces).

Defining a Polygonal Mesh (2)

- It has a square floor one unit on a side.
- Because the barn has flat walls, there are only 7 distinct normal vectors involved, the normal to each face as shown.



Defining a Polygonal Mesh (3)

- The vertex list reports the locations of the distinct vertices in the mesh.
- The list of normals reports the directions of the distinct normal vectors that occur in the model.
- The face list indexes into the vertex and normal lists.

Vertex List for the Barn

vertex	x	y	z
0	0	0	0
1	1	0	0
2	1	1	0
3	0.5	1.5	0
4	0	1	0
5	0	0	1
6	1	0	1
7	1	1	1
8	0.5	1.5	10
9	0	1	1

Normal List for the Barn

- The normal list (as unit vectors, to the 7 basic planes or polygons).

normal	n_x	n_y	n_z
0	-1	0	0
1	-0.707	0.707	0
2	0.707	0.707	0
3	1	0	0
4	0	-1	0
5	0	0	1
6	0	0	-1

Face List for the Barn

Face	**Vertices**	**Normal**
0 (left)	0, 5, 9, 4	0,0,0,0
1 (roof left)	3, 4, 9, 8	1,1,1,1
2 (roof right)	2, 3, 8, 7	2, 2, 2,2
3 (right)	1, 2, 7, 6	3, 3, 3, 3
4 (bottom)	0, 1, 6, 5	4, 4, 4, 4
5 (front)	5, 6, 7, 8, 9	5, 5, 5, 5, 5
6 (back)	0, 4, 3, 2, 1	6, 6, 6, 6, 6

Defining a Polygonal Mesh (4)

- Only the indices of the vertices and normals are used.
- The list of vertices for a face begins with any vertex in the face, and then proceeds around the face vertex by vertex until a complete circuit has been made.
  - There are two ways to traverse a polygon: clockwise and counterclockwise. For instance, face #5 above could be listed as (5, 6, 7, 8, 9) or (9, 8, 7, 6, 5).
  - Convention: ***Traverse the polygon counterclockwise as seen from outside the object*.**

Defining a Polygonal Mesh (5)

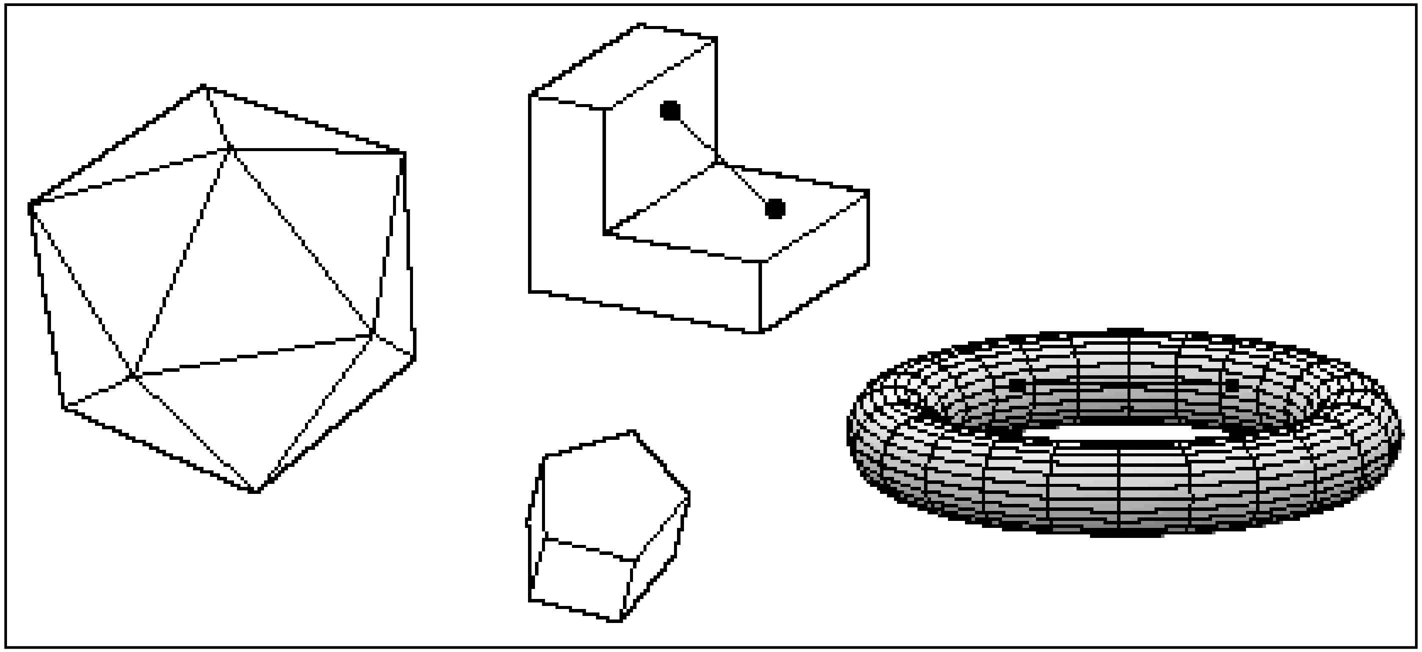
- Using this order, if you traverse around the face by walking from vertex to vertex, the inside of the face is on your left.
- Using the convention allows algorithms to distinguish with ease the front from the back of a face.
- If we use an underlying smooth surface, such as a cylinder, normals are computed for that surface.

Properties of Meshes

- A closed mesh represents a solid object (which encloses a volume).
- A mesh is connected if there is an unbroken path along the edges of the mesh between any two vertices.
- A mesh is simple if it has no holes. Example: a sphere is simple; a torus is not.
- A mesh is planar if every face is a plane polygon. Triangular meshes are frequently used to enforce planarity.

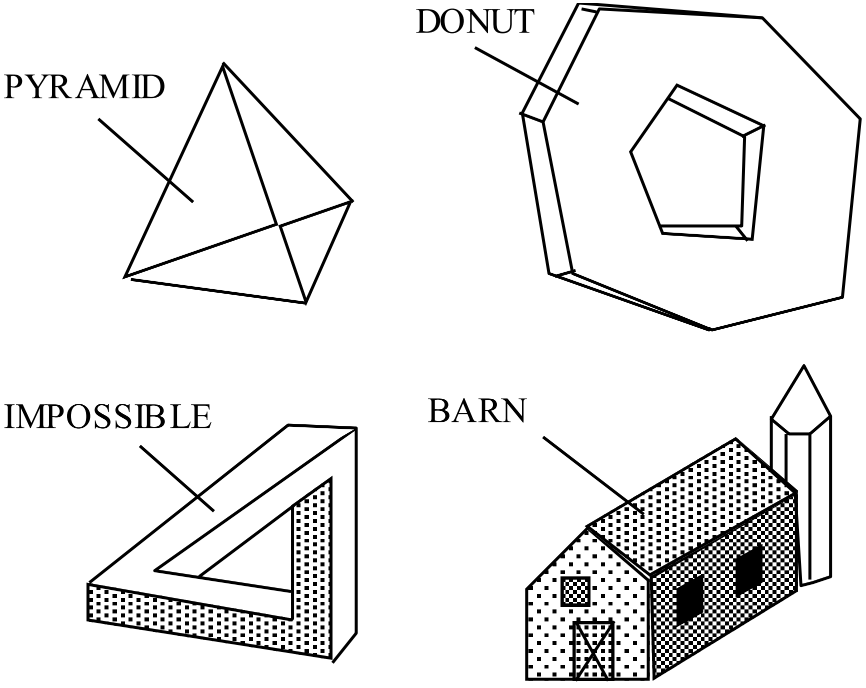
Properties of Meshes (2)

- A mesh is convex if the line connecting any two interior points is entirely inside the mesh.
- Exterior connecting lines are shown for non-convex objects below (step and torus).

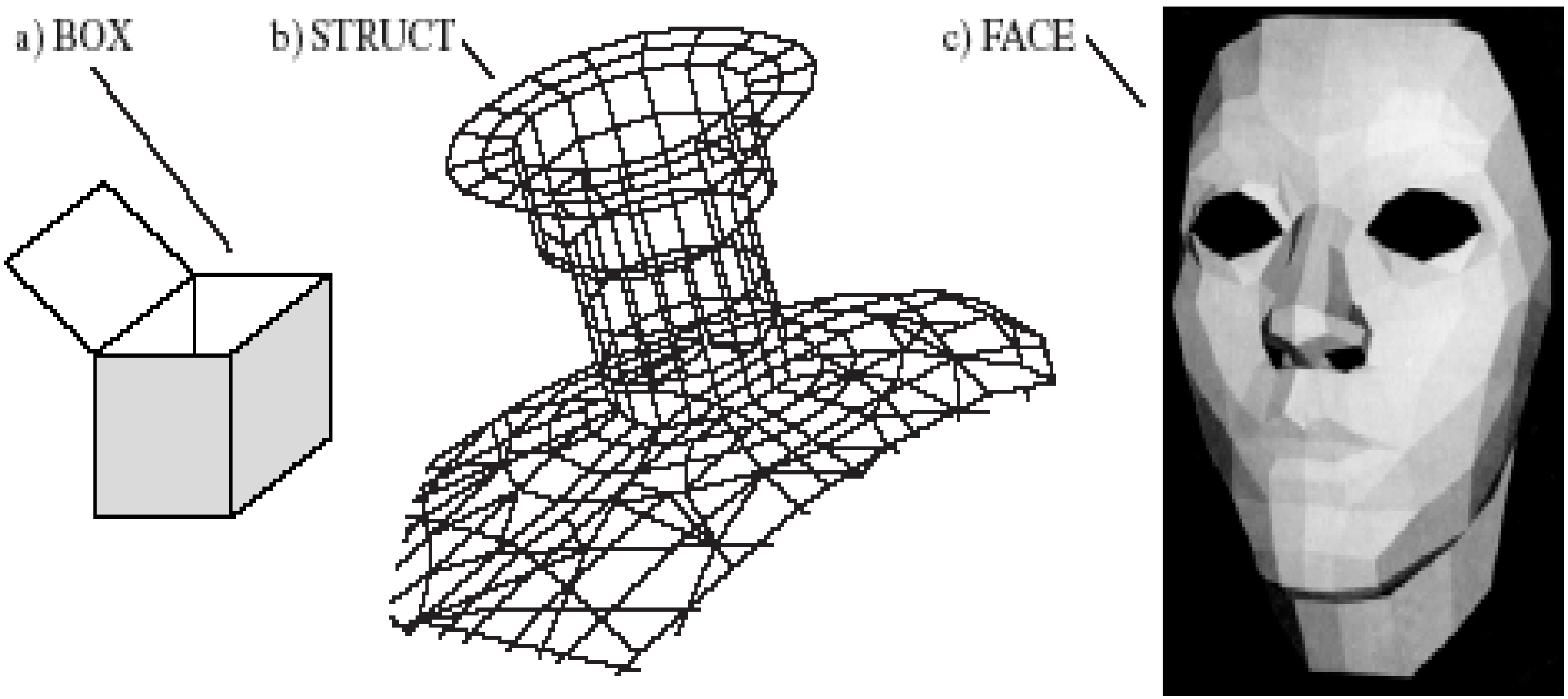


Meshes for Drawing Non-physical Objects

- The figure labeled IMPOSSIBLE looks impossible but is not.
- This object can be represented by a mesh.
- Gershon Elber's web site (http://www.cs.technion.ac.il/~gershon/EscherForReal/) presents a collection of physically impossible objects, and describes how they can be modeled and drawn.



“Thin-skin” Meshes Representing Non-solid Objects



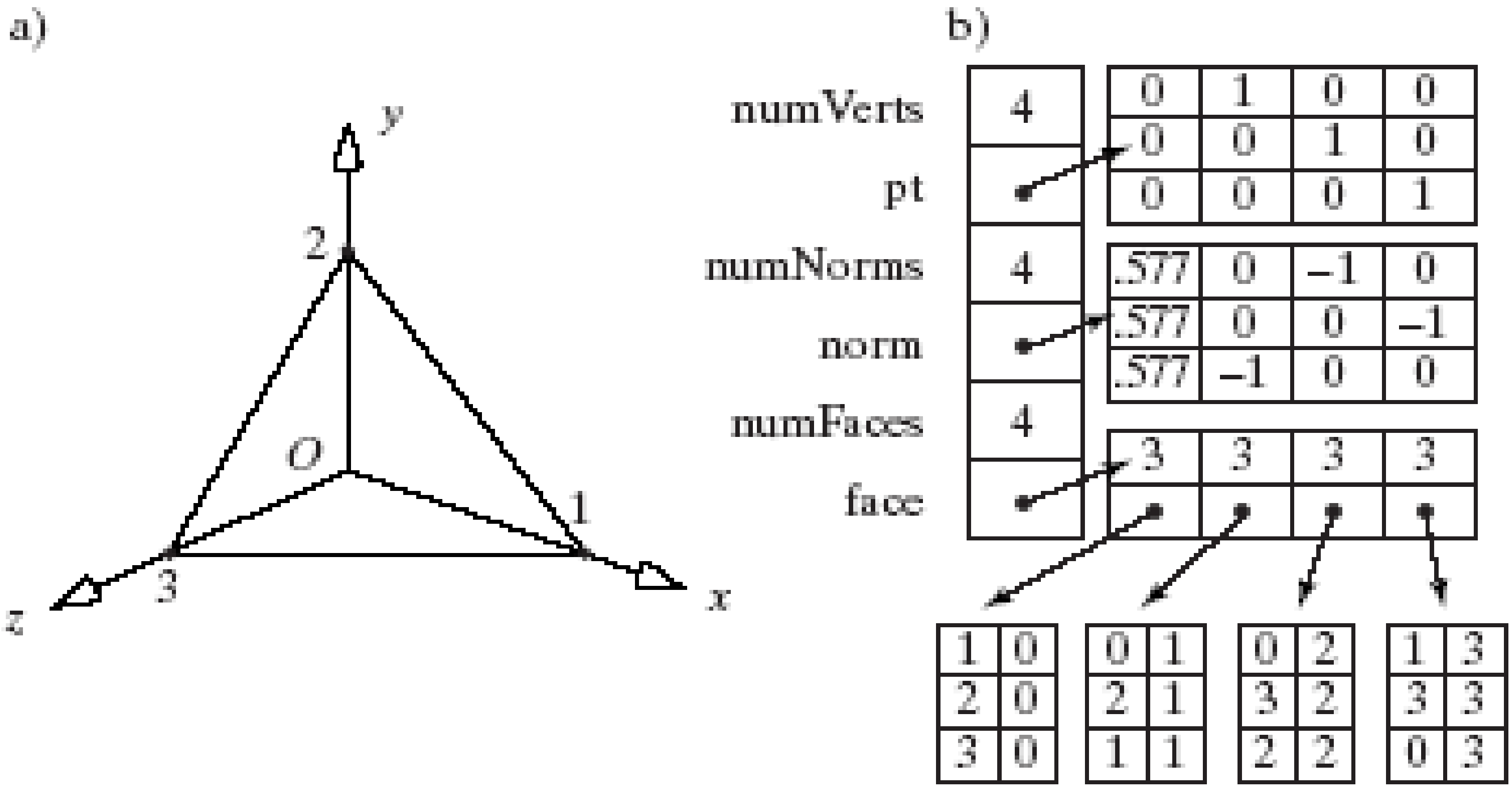
Working with Meshes in a Program

- We want an efficient Mesh class that makes it easy to create and draw the object.
- Since mesh data is frequently stored in a file, we also need simple ways to read and write mesh files.
- Code for classes VertexID, Face, and Mesh is in Fig. 6.15.

Meshes in a Program (2)

- The Face data type is a list of vertices and the normal vector associated with each vertex in the face.
- It is an array of index pairs; the normal to the v^{th} vertex of the f^{th} face has value *norm[face[f].vert[v].normIndex*].
- This indexing scheme is quite orderly and easy to manage, and it allows rapid random access indexing into the pt[ ] array.

Example (tetrahedron & representation)



Drawing the Mesh Object

- Mesh::draw() (Fig. 6.17) traverses the array of faces in the mesh object, and for each face sends the list of vertices and their normals down the graphics pipeline.
- In OpenGL, to specify that subsequent vertices are associated with normal vector **m,** execute glNormal3f (m.x, m.y, m.z).
- For proper shading, these vectors must be normalized. Otherwise, place glEnable(GL_NORMALIZE) in the init() function. This requests that OpenGL automatically normalize all normal vectors.

SDL and Meshes

- To use SDL, simply develop the Mesh class from the Shape class (as SDL does for you) and add the method drawOpenGL(). The book's companion web site gives full details on the Shape class and SDL's supporting classes.
- The Scene class that reads SDL files is already set to accept the keyword mesh, followed by the name of the file that contains the mesh description: e.g., mesh pawn.3vn

Using SDL to Create and Draw Meshes

- The mesh data are in a file with suffix .3vn.
- The first line of the file lists the number of vertices, number of normals, and number of faces, separated by whitespace.
- The second line begins the list of vertices, giving their x, y and z coordinates separated by whitespace.
  - Multiple vertex coordinates may be listed on a single line.

Using SDL (2)

- After all vertices have been listed, the list of normals begins. A normal is specified by nx, ny, and nz, separated by whitespace.
  – Multiple normal values may be listed on a single line.
- The list of faces follows. A face is specified by the number of vertices it has, the list of vertex indices (in counter-clockwise order from outside), and the list of normal indices (same order as the vertex indices).

Using SDL (3)

- We can also use the matrix manipulation functions of SDL to position and orient the mesh drawing.
- Example:
  - push translate 3 5 4 scale 3 3 3 mesh pawn.3vn pop

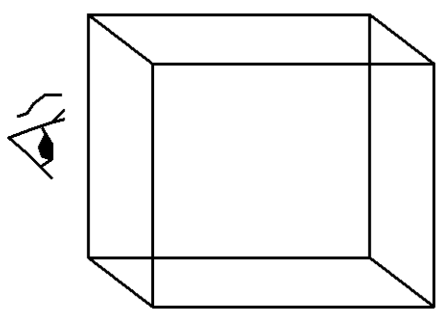
Meshes for Polyhedra

- Polyhedron: connected mesh of simple planar polygons that encloses a finite volume.
  - Every edge is shared by exactly 2 faces.
  - At least 3 edges meet at each vertex.
  - Faces do not interpenetrate. They touch either not at all, or only along their common edge.
  - Euler's formula: $V + F - E = 2$ for a simple polyhedron.

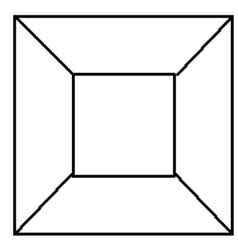
Schlegel Diagrams

- A **Schlegel diagram** reveals the structure of a polygon.
- It views the polyhedron from a point just outside the center of one of its faces.
- The front face appears as a large polygon surrounding the rest of the faces.

a).



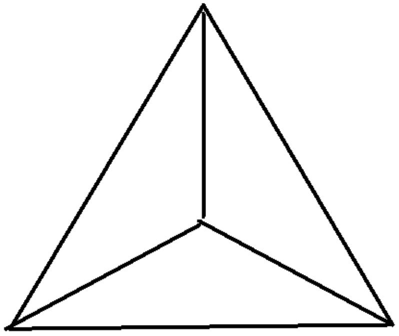
b).



Schlegel Diagrams (2)

- Part a) shows the Schlegel diagram of a pyramid, and parts b) and c) show two different Schlegel diagrams for the basic barn. (Which faces are closest to the eye?).

a).



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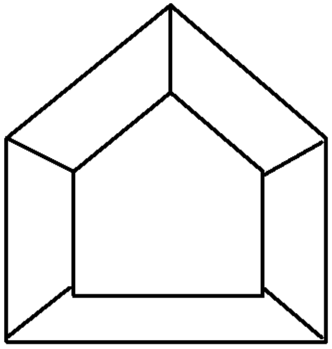
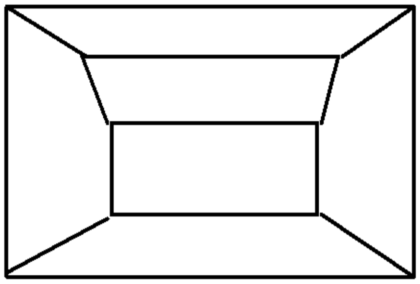
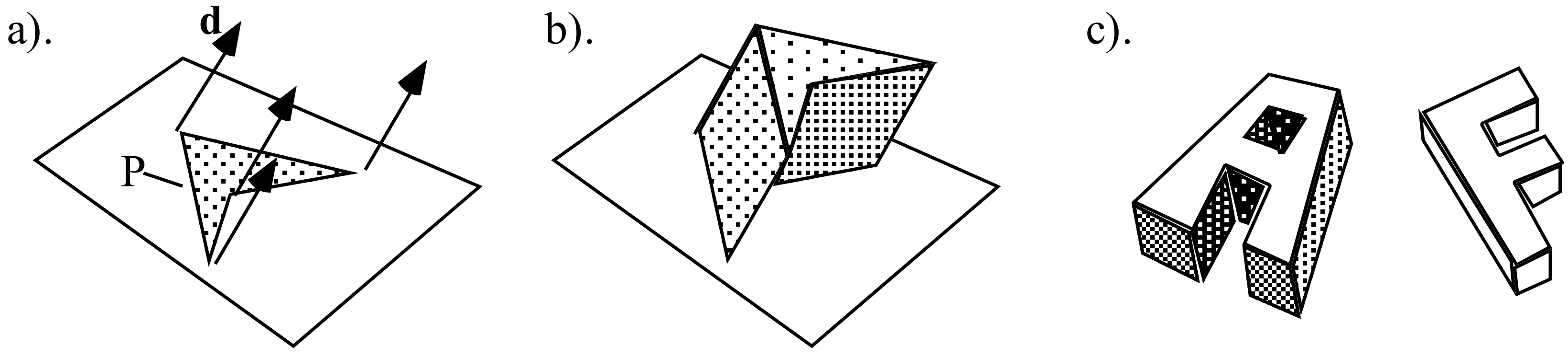


c).



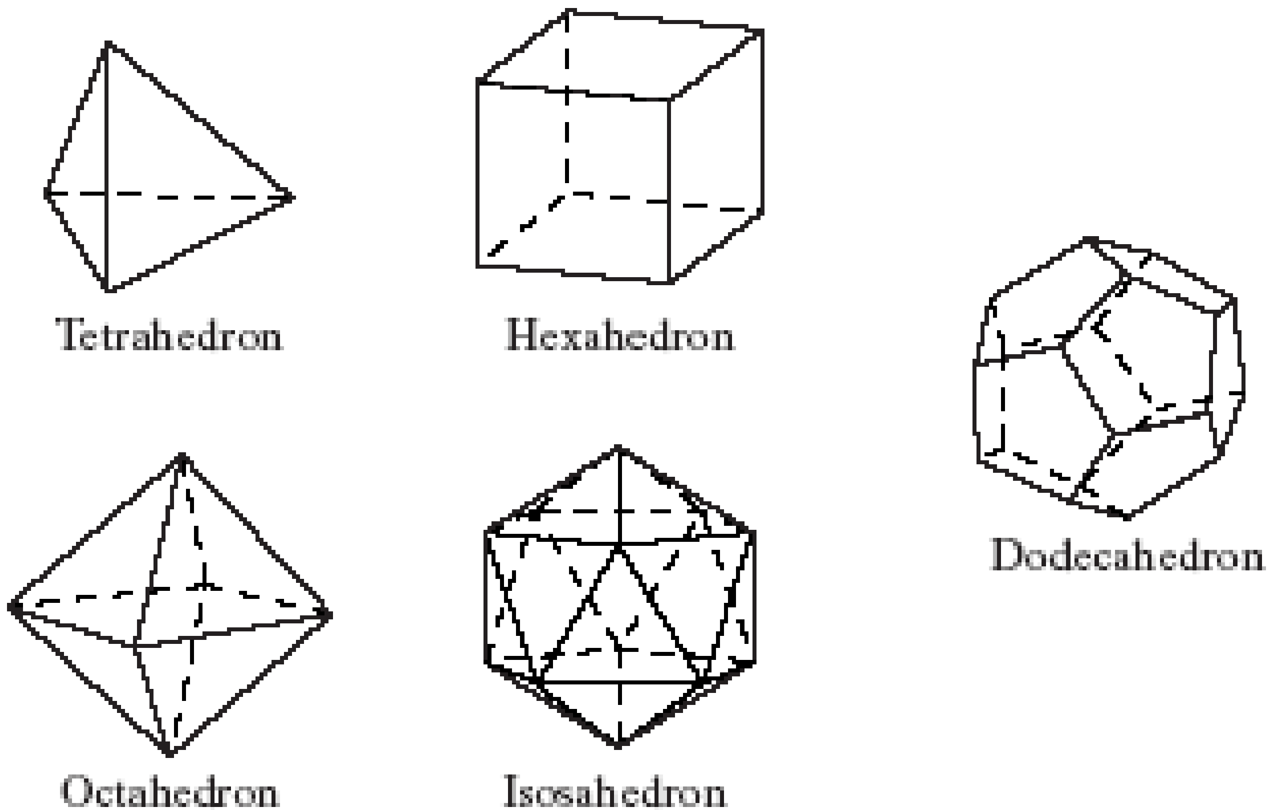
Prisms

- A prism is formed by moving a regular polygon along a straight line.
- When the line is perpendicular to the polygon, the prism is a right prism.



Platonic Solids

- All the faces are identical and each is a regular polygon.

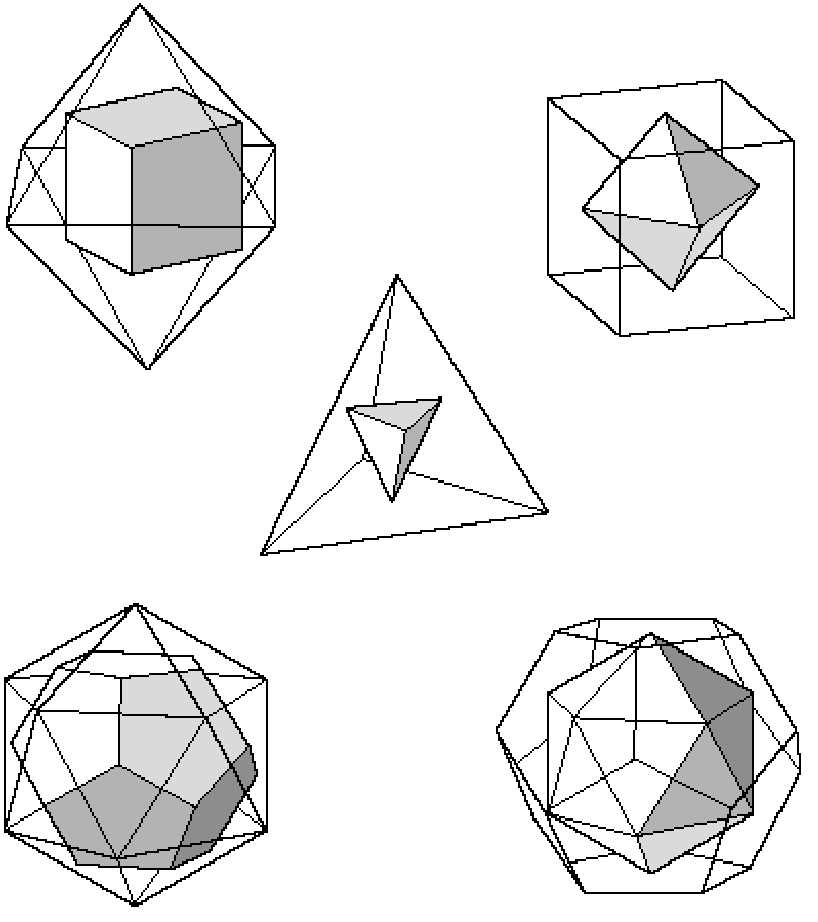


Duals

- Every Platonic solid P has a dual Platonic solid D. The vertices v_i of D are the centers of faces of P, calculated as

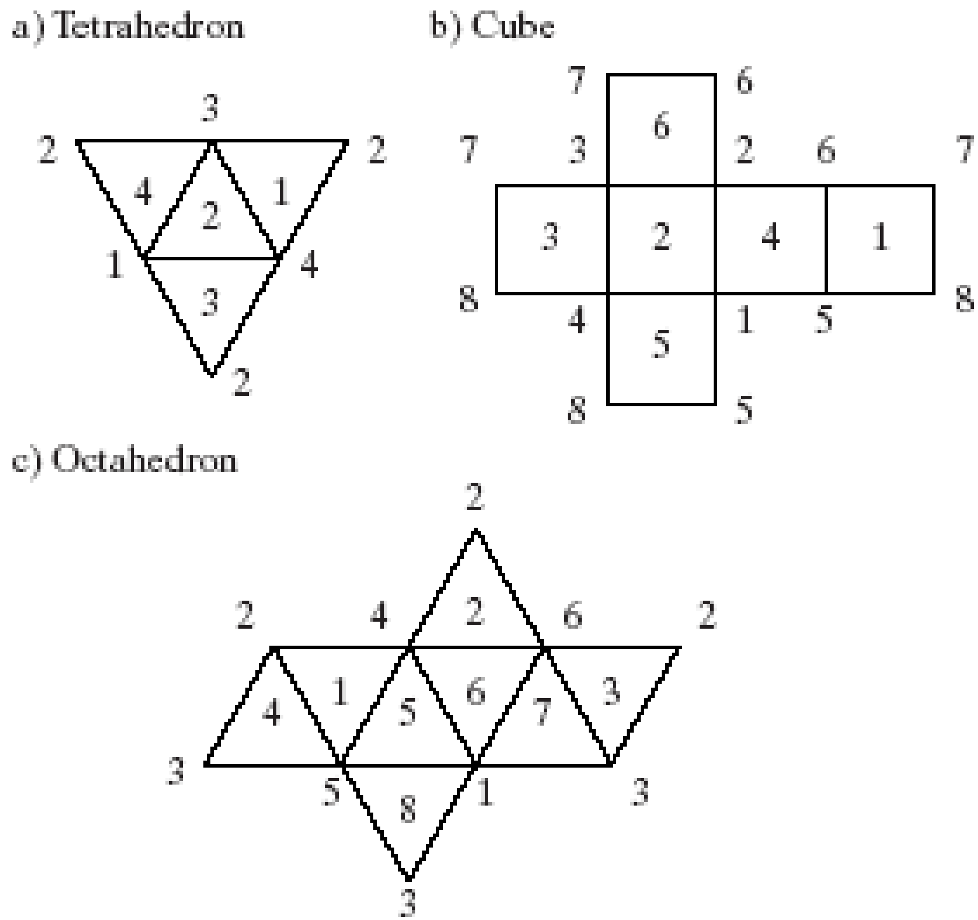
$$c = \frac{1}{n} \sum_{i=0}^{n-1} v_i$$

- The duals are tetrahedron-tetrahedron, hexahedron-octahedron, dodecahedron-icosahedron.



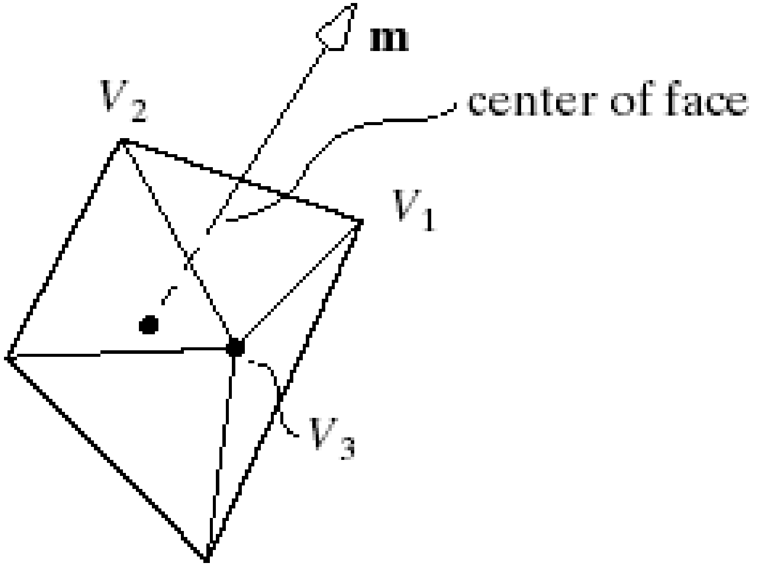
Flattened Models

- To keep track of vertex and face numbering, we use a flat **model**, which is made by cutting along certain edges of each solid and unfolding it to lie flat.



Normal Vectors for the Platonic Solids

- Normals can be found using Newell's method.
- Also, because of the high degree of symmetry of a Platonic solid, **if the solid is centered at the origin**, the normal vector to each face is the vector from the origin to the *center* of the face (the average of the vertices of the face).



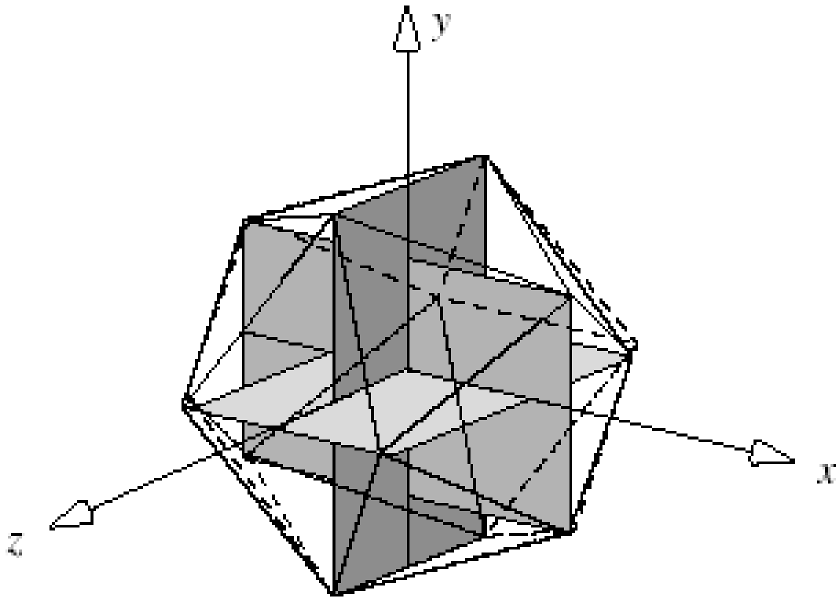
Vertex and Face lists for a Tetrahedron

- For the unit cube having vertices (±1,±1,±1), and the tetrahedron with one vertex at (1,1,1), the tetrahedron has vertex and face lists given below.

Vertex list					Face list	
vertex	x	y	z		Face #	vertices
0	1	1	1		0	1, 2, 3
1	1	-1	-1		1	0, 3, 2
2	-1	-1	1		2	0, 1, 3
3	-1	1	-1		3	0, 2, 1

Icosahedron

- This figure shows that three mutually perpendicular **golden rectangles** inscribe the icosahedron. A vertex list may be read directly from this picture.



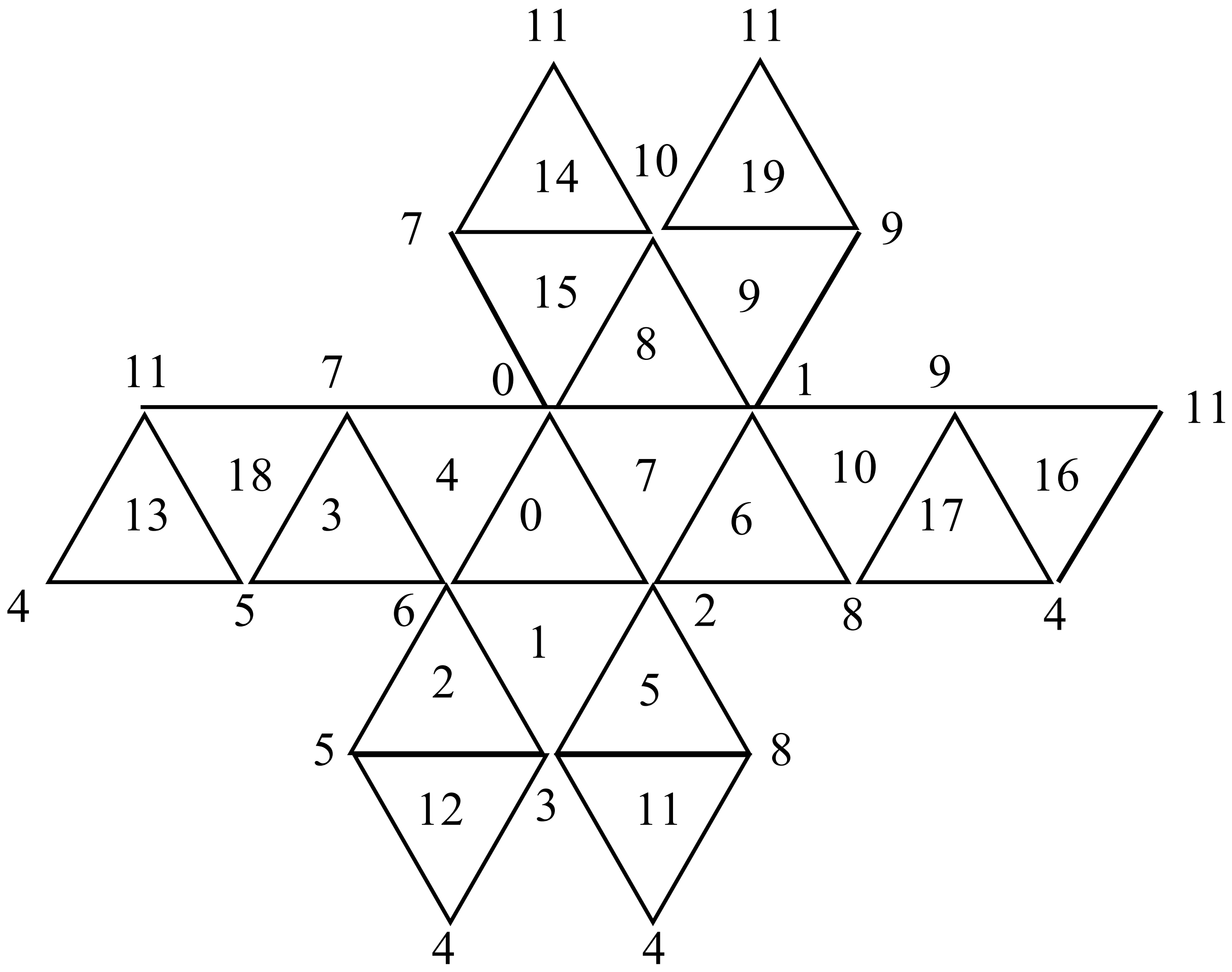
Icosahedron (2)

- We choose to align each golden rectangle with a coordinate axis. For convenience, we define one rectangle so that its longer edge extends from -1 to 1 along the x-axis, and its shorter edge extends from $-\varphi$ to φ, where $\varphi = 0.618$ is the reciprocal of the golden ratio Φ.

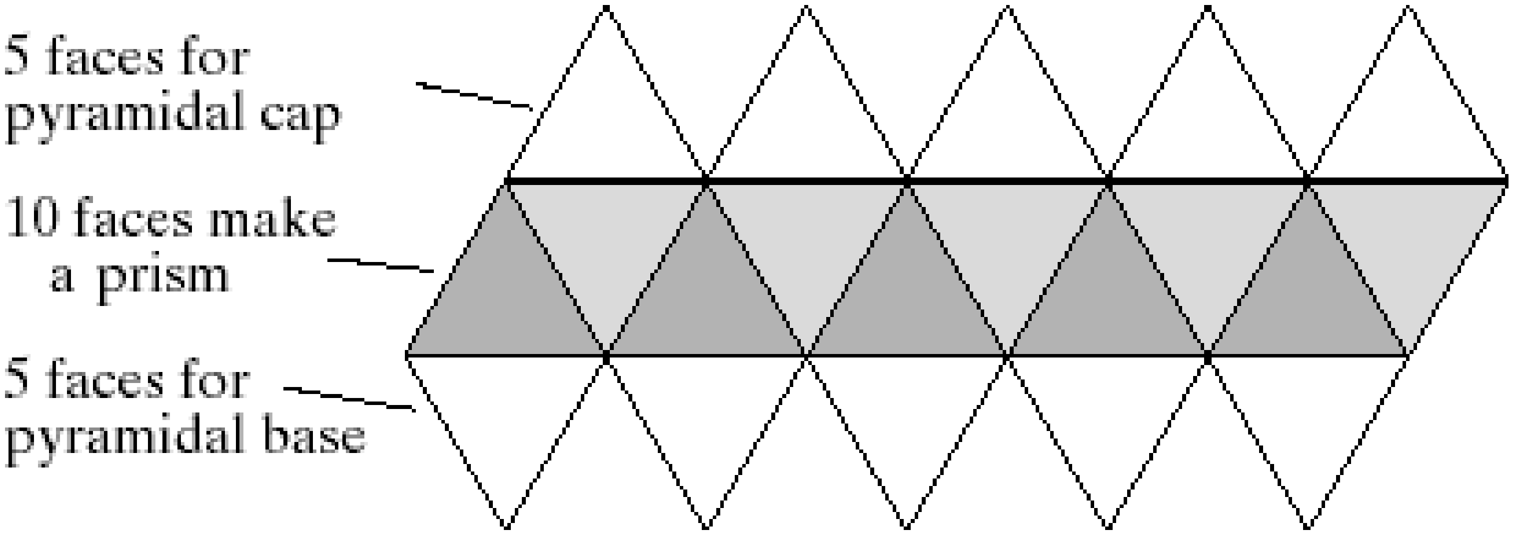
Vertex List for the Icosahedron

Vertex	x	y	z		Vertex	x	y	z
0	0	1	φ		6	φ	0	1
1	0	1	-φ		7	-φ	0	1
2	1	φ	0		8	φ	0	-1
3	1	-φ	0		9	-φ	0	-1
4	0	-1	-φ		10	-1	φ	0
5	0	-1	φ		11	-1	-φ	0

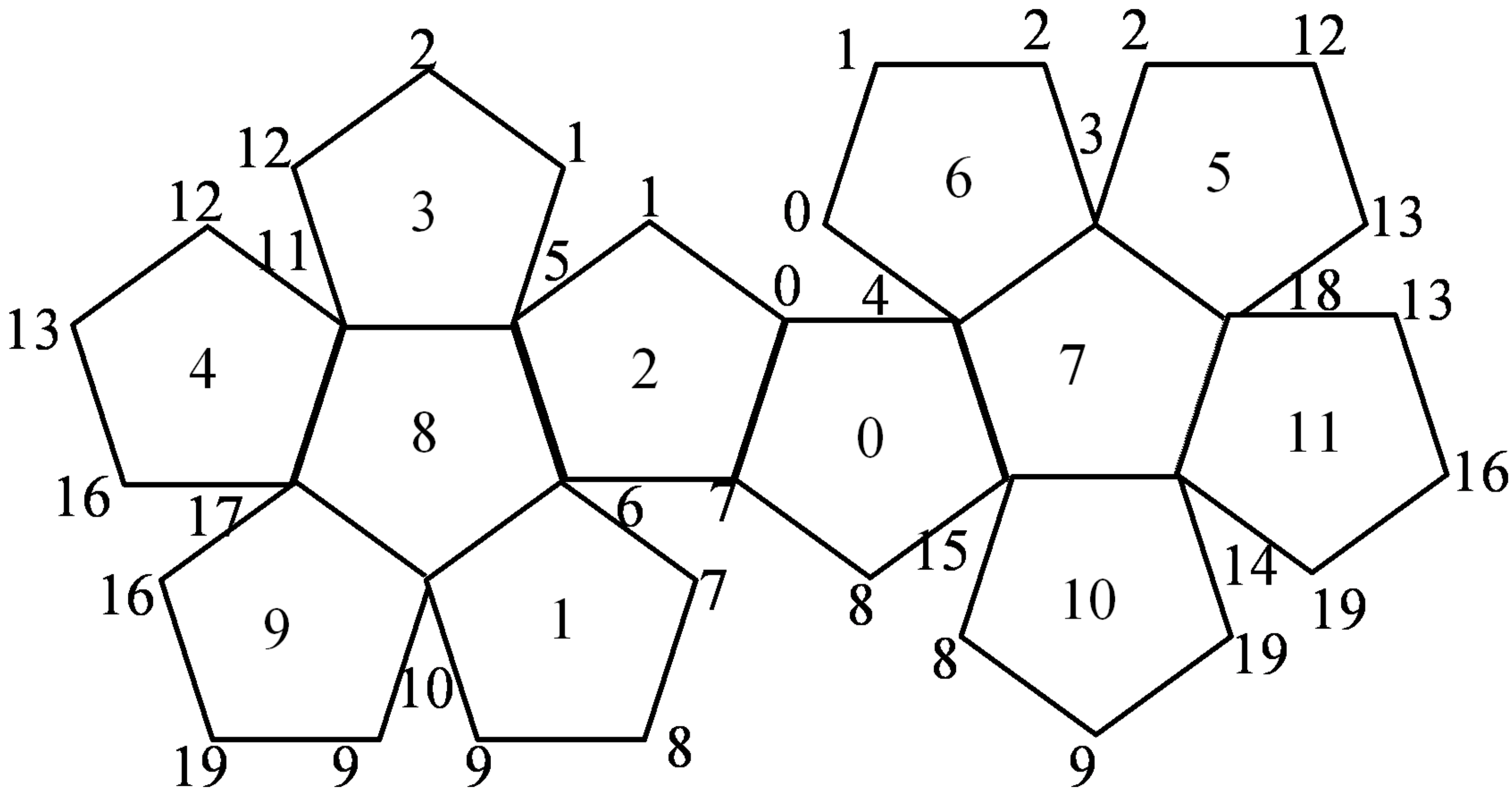
Flattened Model for Icosahedron



Prism Model for Icosahedron

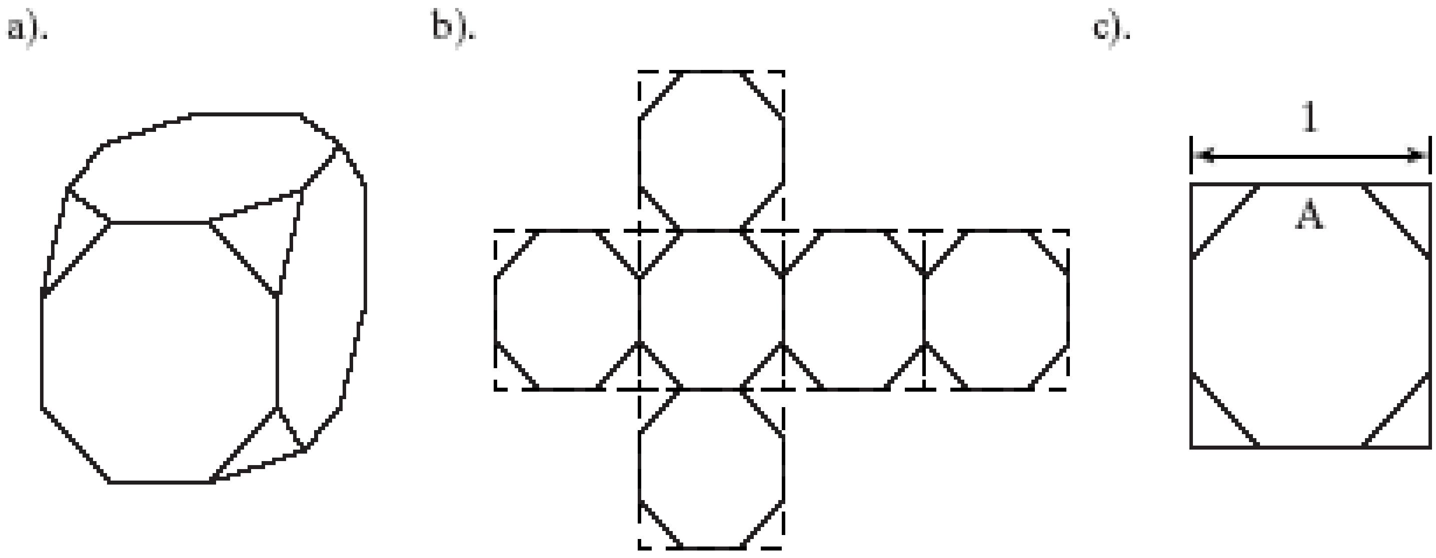


Flattened Model for Dodecahedron



Archimedean Solids

- Have more than one type of polygon as faces; semi-regular.
- Examples: truncated cube (octagon and triangle)



Truncated Cube

- Each edge of the cube is divided into three parts; the middle part of length $A = \frac{1}{\left(1+\sqrt{2}\right)}$ and the middle portion of each edge is joined to its neighbors.
- Thus if an edge of the cube has endpoints *C* and *D*, two new vertices, *V* and *W*, are formed as the affine combinations

$$V = \frac{1+A}{2}C + \frac{1-A}{2}D \quad W = \frac{1-A}{2}C + \frac{1+A}{2}D$$

Number of Archimedean Solids

- Given the constraints that faces must be regular polygons, and that they must occur in the same arrangement about each vertex, there are only 13 possible Archimedean solids.
- Archimedean solids have sufficient symmetry that the normal vector to each face is found using the center of the face.

Truncated Icosahedron

- The truncated icosahedron (soccer ball) consists of regular hexagons and pentagons.
- More recently this shape has been named the **Buckyball** after Buckminster Fuller, because of his interest in geodesic structures similar to this.
- Crystallographers have discovered that 60 atoms of carbon can be arranged at the vertices of the truncated icosahedron, producing a new kind of carbon molecule that is neither graphite nor diamond.
- The material has been named **Fullerene**.

The Buckyball and Flattened Version (Partial)